Local Model Checking of Weighted CTL with Upper-Bound Constraints

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Introduction

- Model checking *both* functional and quantitative properties.
  - Embedded systems - resources are very limited.
  - Resource constraints: cost, memory, bandwidth, power, etc.
- We extend well-known models and temporal logic:
  - *Weighted* CTL & weighted Kripke structures.
- Efficient model checking of WCTL:
  - *Symbolic* dependency graphs
  - Local/on-the-fly fixed-point algorithm

Outline

- Weighted Model Checking
- Dependency graphs
- Symbolic dependency graphs
- Experiments
- Conclusion

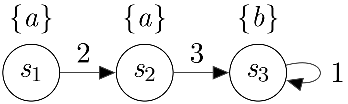
Weighted Kripke Structure

Definition (WKS)

A WKS is a tuple $\mathcal{K} = (S, \mathcal{AP}, L, \rightarrow)$, where

- S is a finite set of states,
- $\mathcal{AP}$ is a set of atomic propositions,
- $L : S \rightarrow \mathcal{P}(\mathcal{AP})$ is a labelling function, and
- $\rightarrow \subseteq S \times \mathbb{N}_0 \times S$ is a transition relation.

Example



Weighted Computation Tree Logic (WCTL)

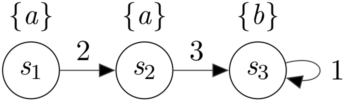
The set of WCTL formulas is given as follows.

$$
\begin{aligned}
\varphi ::= &\ \textit{true} \mid \textit{false} && \text{(Boolean Properties)} \\
&\mid a && \text{(Atomic Proposition)} \\
&\mid \varphi_1 \wedge \varphi_2 && \text{(Conjunction)} \\
&\mid \varphi_1 \vee \varphi_2 && \text{(Disjunction)} \\
&\mid E\ \varphi_1\ U_{\leq k}\ \varphi_2 && \text{(Existential Until)} \\
&\mid A\ \varphi_1\ U_{\leq k}\ \varphi_2 && \text{(Universal Until)} \\
&\mid EX_{\leq k}\ \varphi && \text{(Existential Next)} \\
&\mid AX_{\leq k}\ \varphi && \text{(Universal Next)}
\end{aligned}
$$

where $k \in \mathbb{N}_0$ and $a \in \mathcal{AP}$.

Semantics of the Until Modality

Example



We have that

$$s_1 \models E\ a\ U_{\leq 8}\ b$$
$$s_1 \not\models E\ a\ U_{\leq 4}\ b$$

Consider the only run

$$\sigma = \underbrace{\overbrace{s_1 \xrightarrow{2} s_2}^{a \text{ holds}} \xrightarrow{3} \overset{b \text{ holds}}{s_3}}_{\text{Accumulated weight } 2+3=5} \xrightarrow{1} s_3 \ldots$$

Dependency Graph (1)

Definition (Dependency Graph)

A DG is a pair $G = (V, E)$, where

- V is a set of configurations, and
- $E \subseteq V \times \mathcal{P}(V)$ is a set of hyper-edges.

- An assignment is a mapping $A : V \rightarrow \{1, 0\}$
- A_{min} is the minimum fixed-point assignment.

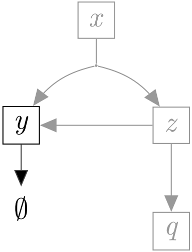
$A_{min}(u) = 1$ if there is $(u, T) \in E$ s.t. for all $v \in T$ we have $A_{min}(v) = 1$.

Functor

$$F(A)(u) = \bigvee_{(u,T)\in E} \left(\bigwedge_{v \in T} A(v) \right)$$

$A_{min} = F(F(\ldots F(A_0)))$ where $A_0(v) = 0$

Example



$A_{min}(y) = 1$ as $(y, \emptyset) \in E$

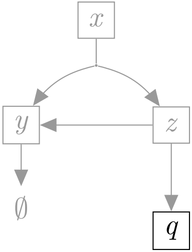
Dependency Graph (2)

Definition (Dependency Graph)

A DG is a pair $G = (V, E)$, where

- V is a set of configurations, and
- $E \subseteq V \times \mathcal{P}(V)$ is a set of hyper-edges.

Example



$$A_{min}(q) = 0$$

- An assignment is a mapping $A : V \to \{1, 0\}$
- A_{min} is the minimum fixed-point assignment.

$A_{min}(u) = 1$ if there is $(u, T) \in E$ s.t.
for all $v \in T$ we have $A_{min}(v) = 1$.

Functor

$$F(A)(u) = \bigvee_{(u,T) \in E} \left(\bigwedge_{v \in T} A(v) \right)$$

$A_{min} = F(F(\ldots F(A_0)))$ where $A_0(v) = 0$

Dependency Graph (3)

Definition (Dependency Graph)

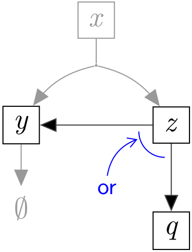
A DG is a pair $G = (V, E)$, where

- V is a set of configurations, and
- $E \subseteq V \times \mathcal{P}(V)$ is a set of hyper-edges.

- An assignment is a mapping $A : V \to \{1, 0\}$
- A_{min} is the minimum fixed-point assignment.

$A_{min}(u) = 1$ if there is $(u, T) \in E$ s.t.
for all $v \in T$ we have $A_{min}(v) = 1$.

Example



$$A_{min}(z) = A_{min}(y) \vee A_{min}(q)$$

Functor

$$F(A)(u) = \bigvee_{(u,T)\in E} \left(\bigwedge_{v\in T} A(v) \right)$$

$A_{min} = F(F(\ldots F(A_0)))$ where $A_0(v) = 0$

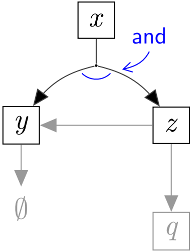
Dependency Graph (4)

Definition (Dependency Graph)

A DG is a pair $G = (V, E)$, where

- V is a set of configurations, and
- $E \subseteq V \times \mathcal{P}(V)$ is a set of hyper-edges.

Example



$$A_{min}(x) = A_{min}(y) \wedge A_{min}(z)$$

- An assignment is a mapping $A : V \to \{1, 0\}$
- A_{min} is the minimum fixed-point assignment.

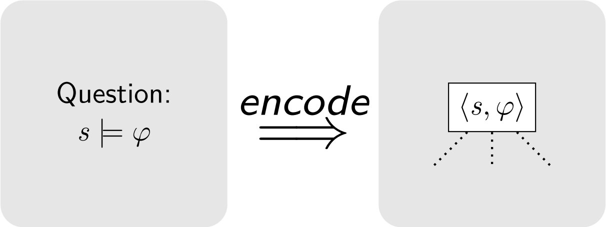
$A_{min}(u) = 1$ if there is $(u, T) \in E$ s.t.
for all $v \in T$ we have $A_{min}(v) = 1$.

Functor

$$F(A)(u) = \bigvee_{(u,T)\in E} \left(\bigwedge_{v \in T} A(v) \right)$$

$A_{min} = F(F(\dots F(A_0)))$ where $A_0(v) = 0$

WCTL Model Checking with Dependency Graphs



Theorem 2

$$s \models \varphi \quad \Leftrightarrow \quad A_{min}(\langle s, \varphi \rangle) = 1$$

Encoding Example ($\varphi =$ ***true***)

Question:
$s \models$ *true*

$\xRightarrow{\textit{encode}}$

$\boxed{\langle s, \textit{true} \rangle}$
$\downarrow$
$\emptyset$

We have the vacuous case, $A_{min}(u) = 1$ *for all* u *in* $\emptyset$*, hence*

$$A_{min}(\langle s, \textbf{true} \rangle) = 1$$

Encoding Example ($\varphi = \textit{\textbf{false}}$)

Question:
$s \models \textit{false}$

$$\xRightarrow{\textit{encode}}$$

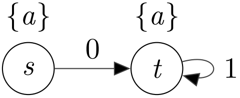
$\boxed{\langle s, \textit{false} \rangle}$

We have the trivial case, as $\langle s, \textit{false} \rangle$ has no hyper-edges, hence

$$A_{min}(\langle s, \textit{\textbf{false}} \rangle) = 0$$

Model Checking Example

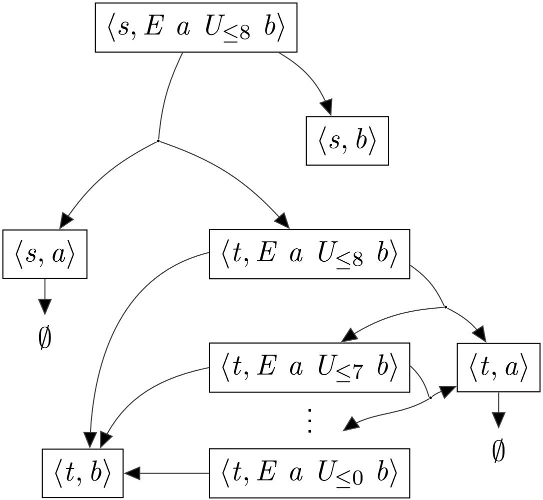
If we take the WKS



and want to determine if

$$s \models E\ a\ U_{\leq 8}\ b$$

we can encode this as:



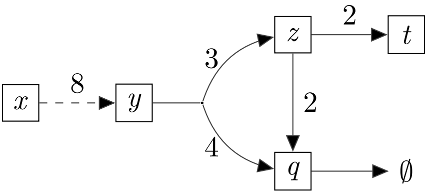
Symbolic Dependency Graphs

Definition (Symbolic Dependency Graphs)

An SDG is a triple $G = (V, H, C)$, where

- V is a finite set of configurations,
- $H \subseteq V \times \mathcal{P}(\mathbb{N}_0 \times V)$ is a finite set of hyper-edges, and
- $C \subseteq V \times \mathbb{N}_0 \times V$ is a finite set of cover-edges.

Example



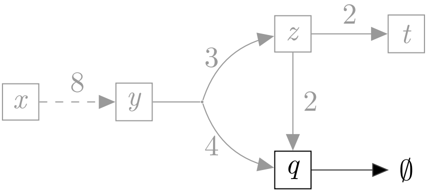
Fixed-Point A_{min} of an SDG (1)

An assignment is a mapping $A : V \rightarrow \mathbb{N}_0 \cup \{\infty\}$
Functor for minimum fixed-point A_{min}

$$F(A)(u) = \begin{cases} 0 & \text{if } \exists(u,k,v) \in C \text{ s.t. } A(v) \leq k \\ \min\limits_{(u,T)\in H}\Big(\max\{w + A(v) \mid (w,v) \in T\}\Big) & \text{otherwise.} \end{cases}$$

$A_{min} = F(\dots F(A_0))$ where $A_0(v) = \infty$.

Example



$$A_{min}(q) = 0 \text{ as } (q, \emptyset) \in E$$

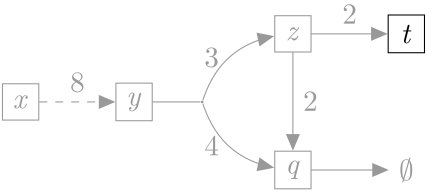
Fixed-Point A_{min} of an SDG (2)

An assignment is a mapping $A : V \to \mathbb{N}_0 \cup \{\infty\}$
Functor for minimum fixed-point A_{min}

$$F(A)(u) = \begin{cases} 0 & \text{if } \exists(u,k,v) \in C \text{ s.t. } A(v) \leq k \\ \min\limits_{(u,T)\in H}\Big(\max\{w + A(v) \mid (w,v) \in T\}\Big) & \text{otherwise.} \end{cases}$$

$A_{min} = F(\dots F(A_0))$ where $A_0(v) = \infty$.

Example



$$A_{min}(t) = \infty$$

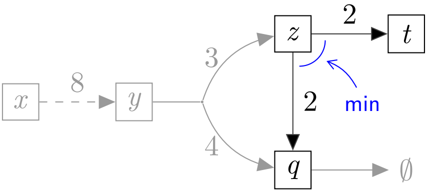
Fixed-Point A_{min} of an SDG (3)

An assignment is a mapping $A : V \to \mathbb{N}_0 \cup \{\infty\}$
Functor for minimum fixed-point A_{min}

$$F(A)(u) = \begin{cases} 0 & \text{if } \exists(u,k,v) \in C \text{ s.t. } A(v) \leq k \\ \min\limits_{(u,T)\in H}\Big(\max\{w + A(v) \mid (w,v) \in T\}\Big) & \text{otherwise.} \end{cases}$$

$A_{min} = F(\dots F(A_0))$ where $A_0(v) = \infty$.

Example



$$A_{min}(z) = \min(2 + A_{min}(q), 2 + A_{min}(t))$$

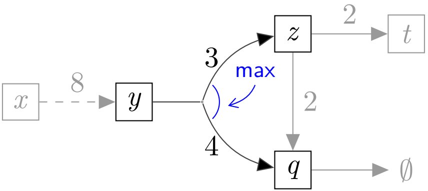
Fixed-Point A_{min} of an SDG (4)

An assignment is a mapping $A : V \to \mathbb{N}_0 \cup \{\infty\}$
Functor for minimum fixed-point A_{min}

$$F(A)(u) = \begin{cases} 0 & \text{if } \exists(u,k,v) \in C \text{ s.t. } A(v) \leq k \\ \min\limits_{(u,T)\in H} \big(\max\{w + A(v) \mid (w,v) \in T\}\big) & \text{otherwise.} \end{cases}$$

$A_{min} = F(\dots F(A_0))$ where $A_0(v) = \infty$.

Example



$$A_{min}(y) = \max(3 + A_{min}(z), 4 + A_{min}(q))$$

Fixed-Point A_{min} of an SDG (5)

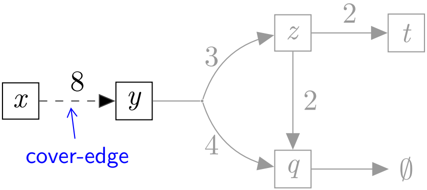
An assignment is a mapping $A : V \to \mathbb{N}_0 \cup \{\infty\}$

Functor for minimum fixed-point A_{min}

$$F(A)(u) = \begin{cases} 0 & \text{if } \exists(u,k,v) \in C \text{ s.t. } A(v) \leq k \\ \min\limits_{(u,T)\in H} \Big(\max\{w + A(v) \mid (w,v) \in T\} \Big) & \text{otherwise.} \end{cases}$$

$A_{min} = F(\dots F(A_0))$ where $A_0(v) = \infty$.

Example

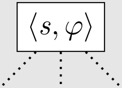


$$A_{min}(x) = \begin{cases} 0 & \text{if } A_{min}(y) \leq 8 \\ \infty & \text{otherwise} \end{cases}$$

WCTL Model Checking with SDGs

Question:
$s \models \varphi$

$\xRightarrow{\textit{encode}}$



Theorem 5

$$s \models \varphi \quad \Leftrightarrow \quad A_{min}(\langle s, \varphi \rangle) = 0$$

Encoding Example ($\varphi =$ ***true***)

Question:
$s \models$ *true*

$$\xRightarrow{\textit{encode}}$$

$$\boxed{\langle s, \textit{true} \rangle} \longrightarrow \emptyset$$

We have the empty target-set and $\max(\emptyset) = 0$, *hence*

$$A_{min}(\langle s, \textbf{true} \rangle) = 0$$

Encoding Example ($\varphi =$ ***false***)

Question:
$s \models$ *false*

$\xRightarrow{\textit{encode}}$

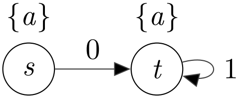
$\boxed{\langle s, \textit{false} \rangle}$

We have the trivial case, as $\langle s, \textit{false} \rangle$ has no hyper-edges, hence

$$A_{min}(\langle s, \textit{false} \rangle) = \infty$$

Model Checking with SDG Example

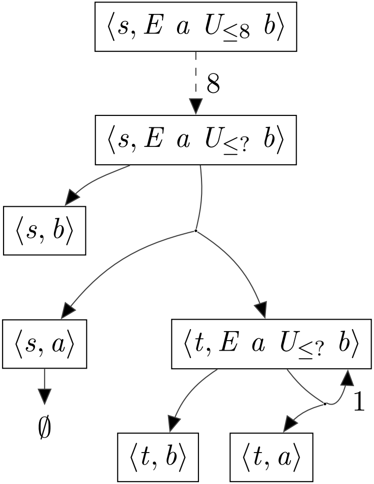
If we take the WKS



and want to determine if

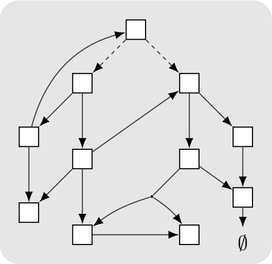
$$s \models E\ a\ U_{\leq 8}\ b$$

then we can encode this as:



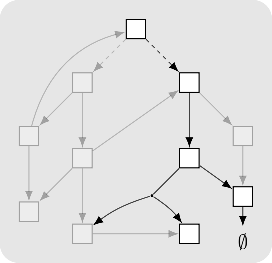
Fixed-Point Algorithms

Global



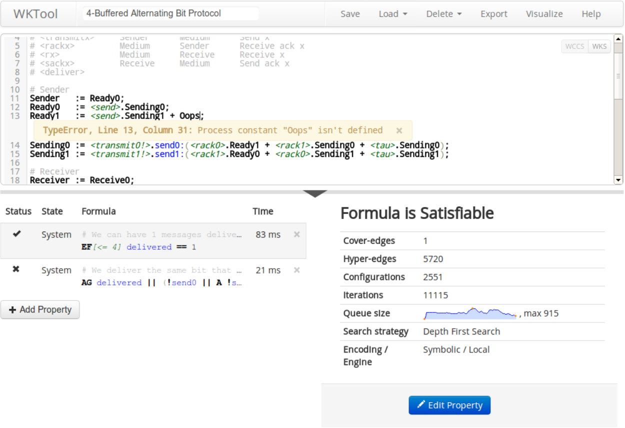
- Up-front construction of SDG.
- Repeated application of F.
- Terminates with A_{min} for all configurations.

Local



- On-the-fly construction of SDG.
- Top-down w. backwards propagation.
- Terminates with A_{min} for the initial configuration.

Model Checking with WKTool



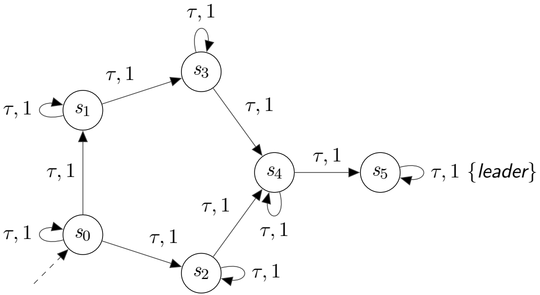
http://wktool.jonasfj.dk/

Experiments

Evaluation of DG vs. SDG and local vs. global for SDG.

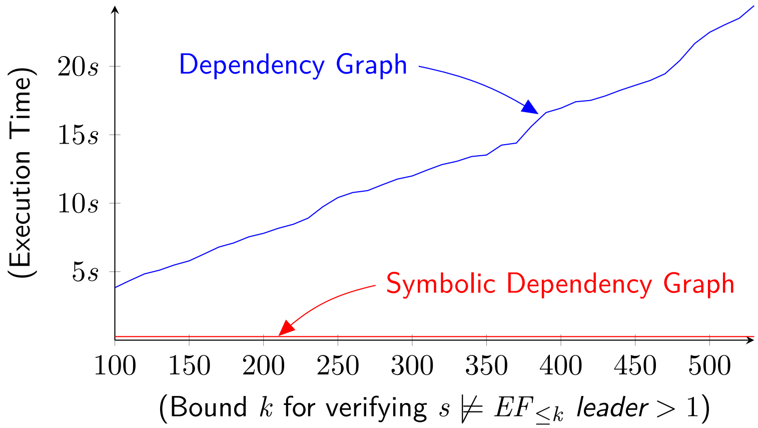
Models:

- Leader Election
- Alternating Bit Protocol
- Task Graph Scheduling problems for 2 processors



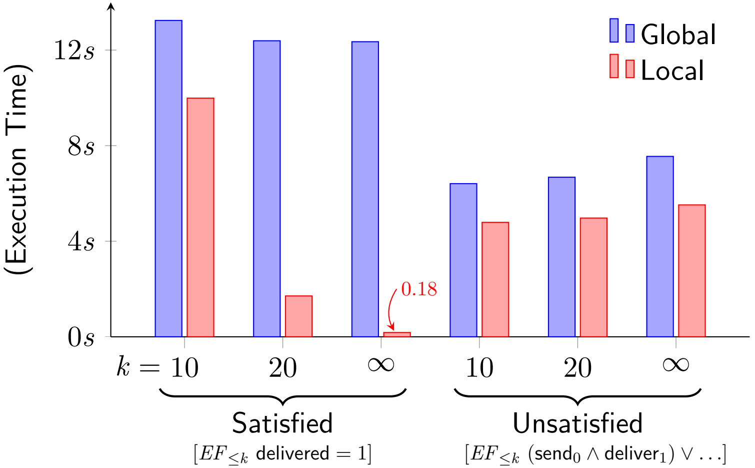
Direct vs. Symbolic (Scaling Bound)

Leader election with DG and SDG encodings using global algorithms.

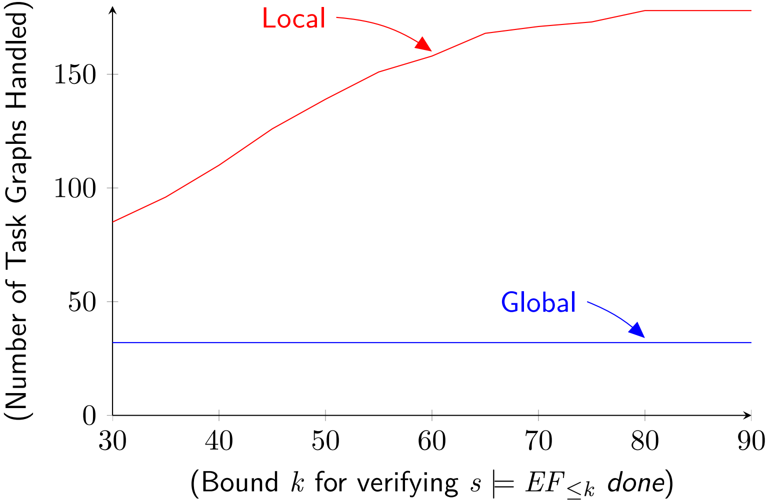


Comparing Global and Local for SDGs

Alternating bit protocol with buffer size 9 (satisfied) and 8 (unsatisfied).

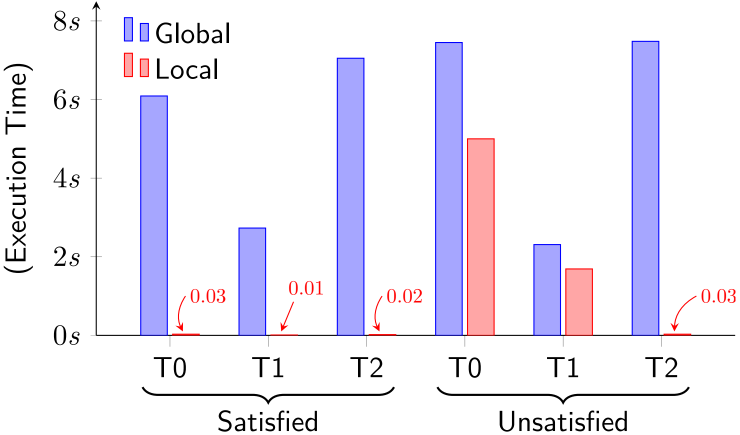


Global vs. Local on 180 Task Graphs



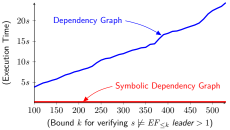
Comparing Global and Local for SDGs

Task graphs T0, T1 and T2 with 5 tasks and nested WCTL properties.



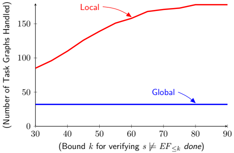
$\left[EF_{\leq 90}\ \left(t_3^{ready} \wedge AF_{\leq 80}\ \textit{done}\right)\right]$ $\left[EF_{\leq 5}\ \left(t_3^{ready} \wedge AF_{\leq 10}\ \textit{done}\right)\right]$

Conclusion



Symbolic Dependency Graphs are advantageous for weighted model checking

Local algorithm can handle larger problems



Future work:

- Alternating fixed-points for *full* WCTL logic.
- Lower-bound constraints on temporal operators.
- Heuristics for search strategy.